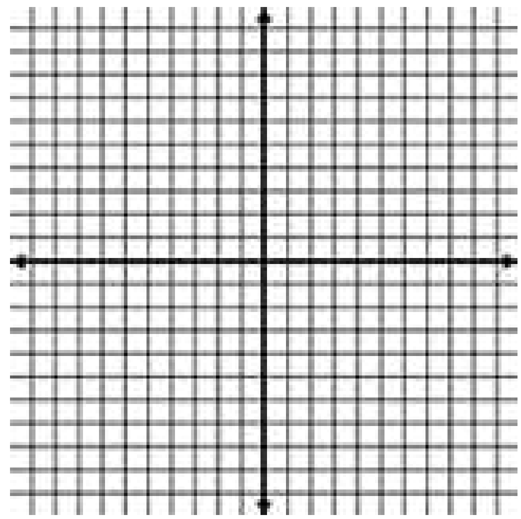
When you are done with your homework you should be able to…

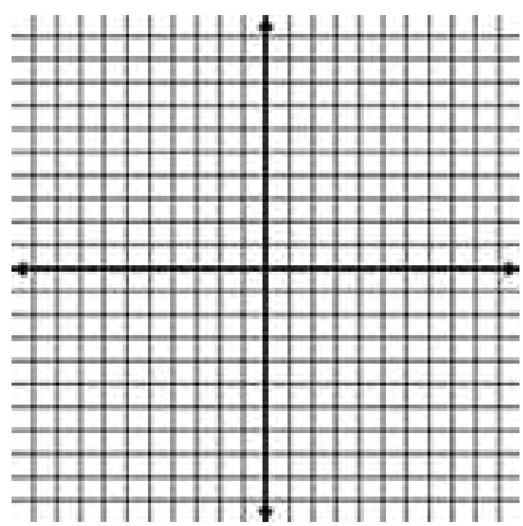
- π Find the volume of a solid of revolution using the disk method
- π Find the volume of a solid of revolution using the washer method
- π Find the volume of a solid with known cross sections

Warm-up: Sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

a. $f(x) = -x^2 + 4x + 1, \;\; g(x) = x + 1$



b. $f(y) = y(2 - y), \;\; g(y) = 0, \;\; y = -1, \;\; y = 2$



THE DISK METHOD

An important application of the ________________________ integral is its use in finding the ______________________ of a three-dimensional solid—one whose ______________________ sections are ______________________.

Solids of revolution are used commonly in engineering and manufacturing. Some examples are _______________, ____________________, ____________, ____________________, and ___________________.

If a ______________________ in the _____________________ is ______________________ about a __________________, the resulting ____________________ is a **solid of revolution**, and the line is called the **axis of revolution**.

THE DISK METHOD

To find the volume of a solid of revolution with the **disk method** use one of the following:

Horizontal Axis of Revolution

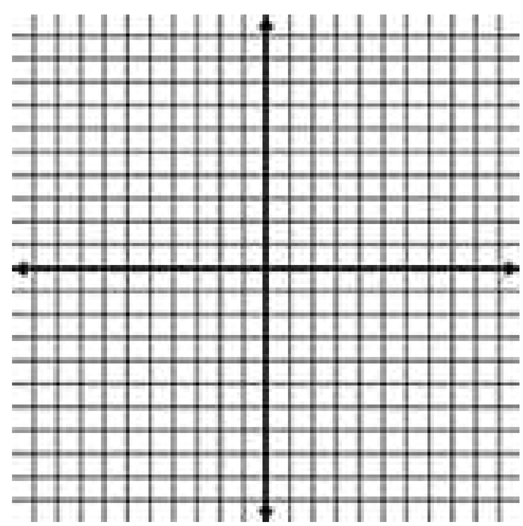
$$V = \pi \int_a^b \left[R(x)\right]^2 dx$$

Vertical Axis of Revolution

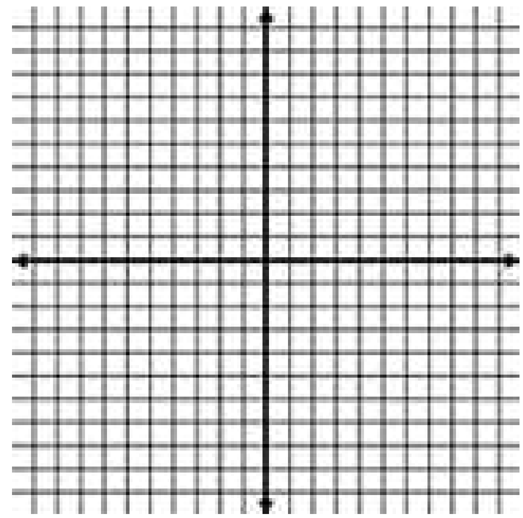
$$V = \pi \int_c^d \left[R(y)\right]^2 dy$$

Example 1: Find the volume of the solids generated by revolving the regions bounded by the graphs of the equations about the given line.

a) $y = 2x^2,\ y = 0,\ x = 2,$ about the $x-$axis.



b) $y = 2x^2,\ y = 0,\ x = 2,$ about the $y-$axis .



THE WASHER METHOD

The disk method can be extended to cover solids of revolution with ___________

by replacing the representative _________________ with a representative

___________________________.

THE WASHER METHOD

To find the volume of a solid of revolution with the **washer method** use one of the following:

Horizontal Axis of Revolution

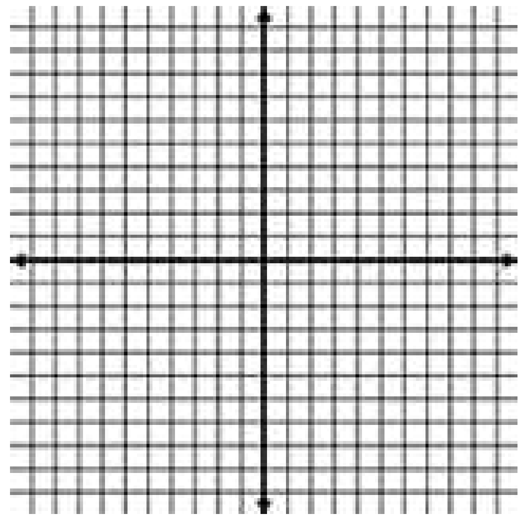
$$V = \pi \int_a^b \left(\left[R(x) \right]^2 - \left[r(x) \right]^2 \right) dx$$

Vertical Axis of Revolution

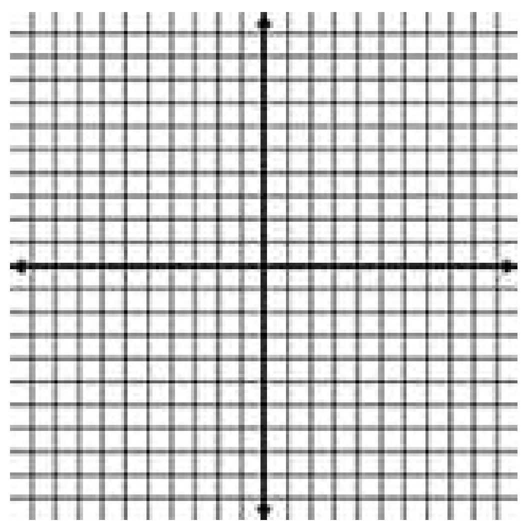
$$V = \pi \int_c^d \left(\left[R(y) \right]^2 - \left[r(y) \right]^2 \right) dy$$

Example 2: Find the volume of the solids generated by revolving the regions bounded by the graphs of the equations about the given line.

a) $y = 2x^2,\ y = 0,\ x = 2,$ about the line $x = 6$.



b) $y = \cos x,\ y = 1,\ x = 0\ , x = \frac{\pi}{2}$ about the line $y = 2$.



SOLIDS WITH KNOWN CROSS SECTIONS

With the disk method, you can find the _______________________ of a solid

having a _____________________ cross section whose area is ____________. This method can be generalized to solids of any shape, as long as you know a formula for the area of an arbitrary cross section. Some common cross sections

are ___________________, ________________________,

_____________________, ________________________, and

______________________________.

VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

1. For cross sections of area $A(x)$ taken perpendicular to the *x*-axis,

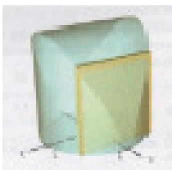
$$V=\int_a^b A(x)dx$$

2. For cross sections of area $A(y)$ taken perpendicular to the *y*-axis,

$$V=\int_c^d A(y)dy$$

Example 3: Find the volumes of the solids whose bases are bounded by the circle $x^2 + y^2 = 4$ with the indicated cross sections taken perpendicular to the *x*-axis.

a) Squares



b) Semicircles